Prime Factorisation, HCFs And LCMs

HCF = Highest Common Factor.
LCM = Lowest Common Multiple.
Patrons are reminded that it is *always* preferable to factories a number into its prime factor form. For example
$3960 \Rightarrow$ BAD,
$2^3 \times 3^2 \times 5 \times 11 \Rightarrow$ GOOD.

1. Express the following numbers as a product of primes
   (a) 60. $\boxed{2^2 \times 3 \times 5}$
   (b) 81. $\boxed{3^4}$
   (c) 53. $\boxed{53}$
   (d) 102. $\boxed{2 \times 3 \times 17}$
   (e) 72. $\boxed{2^3 \times 3^2}$
   (f) 1 485. $\boxed{3^3 \times 5 \times 11}$
   (g) 605. $\boxed{5 \times 11^2}$
   (h) 11 375. $\boxed{5^3 \times 7 \times 13}$
   (i) 230. $\boxed{2 \times 5 \times 23}$
   (j) 8 856. $\boxed{2^3 \times 3^3 \times 41}$
   (k) 91 500. $\boxed{2^2 \times 3 \times 5^3 \times 61}$
   (l) 59 290. $\boxed{2 \times 5 \times 7^2 \times 11^2}$
2. Find the smallest integer that 11340 needs to be multiplied by to make a perfect square.
3. Find the smallest integer that 38808 needs to be multiplied by to make a perfect cube.
4. Find the HCF and LCM of 25 and 30. $\boxed{\text{HCF} = 5, \text{LCM} = 2 \times 3 \times 5^2 = 150}$
5. Find the HCF and LCM of 100 and 110. $\boxed{\text{HCF} = 2 \times 5 = 10, \text{LCM} = 2^2 \times 5^2 \times 11 = 1100}$
6. Find the HCF and LCM of 72 and 90. $\boxed{\text{HCF} = 2 \times 3^2 = 18, \text{LCM} = 2^3 \times 3^2 \times 5 = 360}$
7. Find the HCF and LCM of $2^3 \times 3 \times 7$ and $3^2 \times 5 \times 7$. $\boxed{\text{HCF} = 3 \times 7 = 21, \text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520}$
8. Find the HCF and LCM of $2^2 \times 5^3 \times 11^2$ and $3 \times 5^4 \times 7^2$. $\boxed{\text{HCF} = 5^3 = 125, \text{LCM} = 2^2 \times 3 \times 5^4 \times 7^2 \times 11^2 = 44467500}$
9. (Thanks to JM for this gem.) The LCM of two numbers is 84. If one of the numbers is 28, find all possible values of the other number. $\boxed{3, 6, 12, 21, 42, 84}$
10. (Thanks, again, to JM for this gem.) Find the smallest integer greater than 1 with the property that division by each of 3, 4 and 5 yields a remainder of 1. □
11. (Maclaurin) How many positive integers leave a remainder of 31 when divided into 2011? □

Algebraic Problems

1. Find the HCF and LCM of $3x^2y^7$ and $12x^4y$.
2. Find the HCF and LCM of $6xy^7z$ and $8x^4z^2$.
3. Find the HCF and LCM of $600xyz$ and $750x^2z^3$.
4. Combine into a single fraction $\dfrac{3}{2x^2y} + \dfrac{4}{5xy^4}$.
5. Combine into a single fraction $\dfrac{1}{5x^3yz^2} + 3 - \dfrac{2}{3x^2z^4}$.